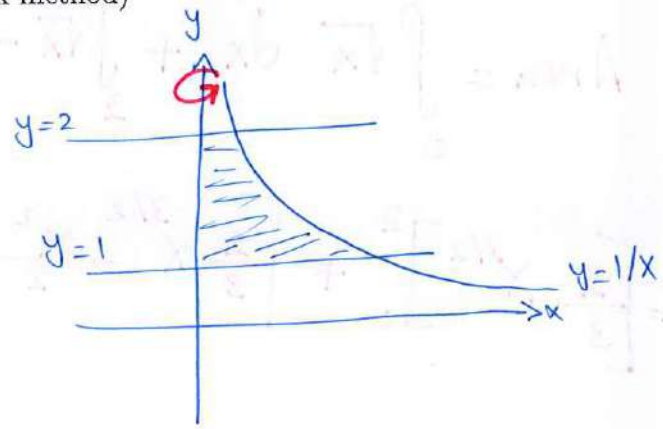
Section (21)

BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT
MATH1411 - QUIZ 3

Name :..... Student Number..... Section 10/10

Question #1: 3.5 pt.
Find the volume of the solid generated by revolving the area enclosed between the curve $y = \frac{1}{x}$,the y-axis and the lines $y = 1$ and $y = 2$ about the y-axis (Disk method)



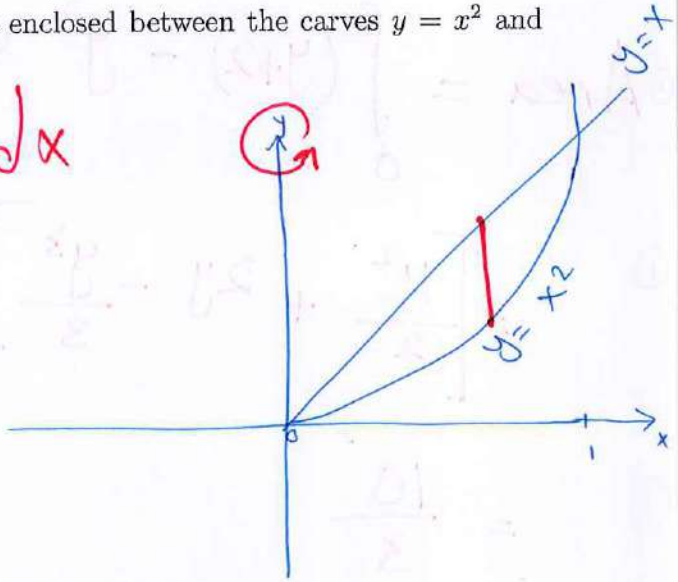
(1) $V = \int_1^2 A(y)\,dy$

(1) $= \int_1^2 \pi \left[\frac{1}{y}\right]^2 dy$

(1) $= \pi \left[\frac{-1}{y}\right]_1^2$

(0.5) $= \frac{\pi}{2}$

Question #2: 3.5 pt
Find the volume of solid generated by revolving the area enclosed between the carves $y = x^2$ and $y = x$ about the y-axis (using shell method)



(1) $V = 2\pi \int_0^1 (\text{shell radius})(\text{shell height})\,dx$

(1) $= 2\pi \int_0^1 (x)(x - x^2)\,dx$

(1) $= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$

(0.5) $= \frac{\pi}{6}$

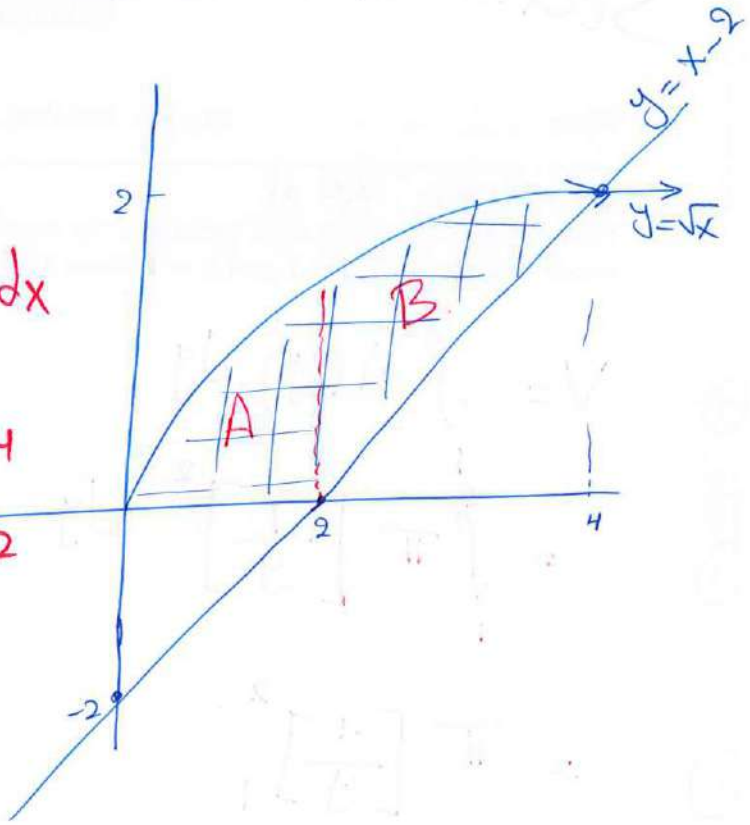
Question #3: 3 Pt.

Find the area enclosed between the curve $x = y^2$, the x-axis, and the line $x = y + 2$

$$\text{Area} = \int_0^2 \sqrt{x}\, dx + \int_2^4 \sqrt{x} - (x-2)\, dx$$

$$= \left[\frac{2}{3}x^{3/2}\right]_0^2 + \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2} + 2x\right]_2^4$$

$$= \frac{10}{3}$$

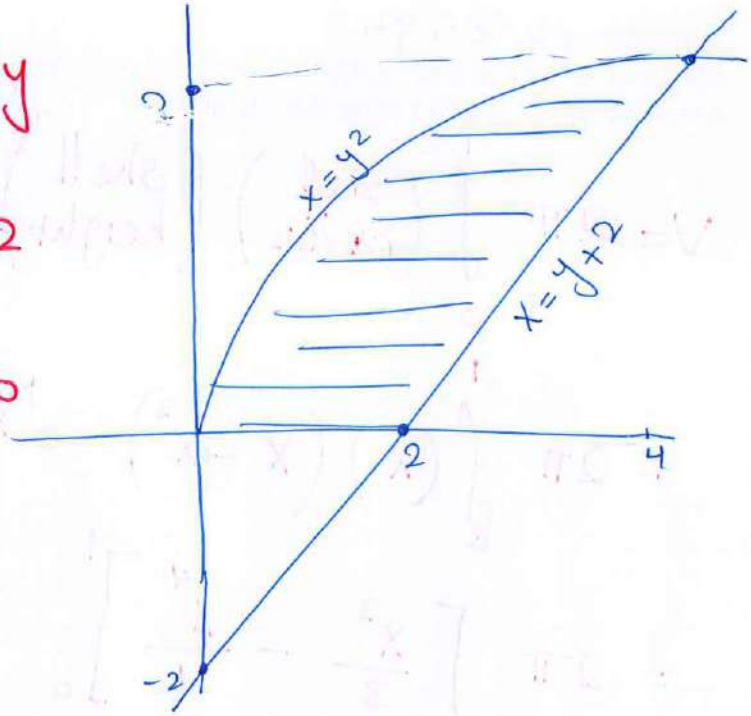


OR

$$\text{Area} = \int_0^2 (y+2) - y^2\, dy$$

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_0^2$$

$$= \frac{10}{3}$$

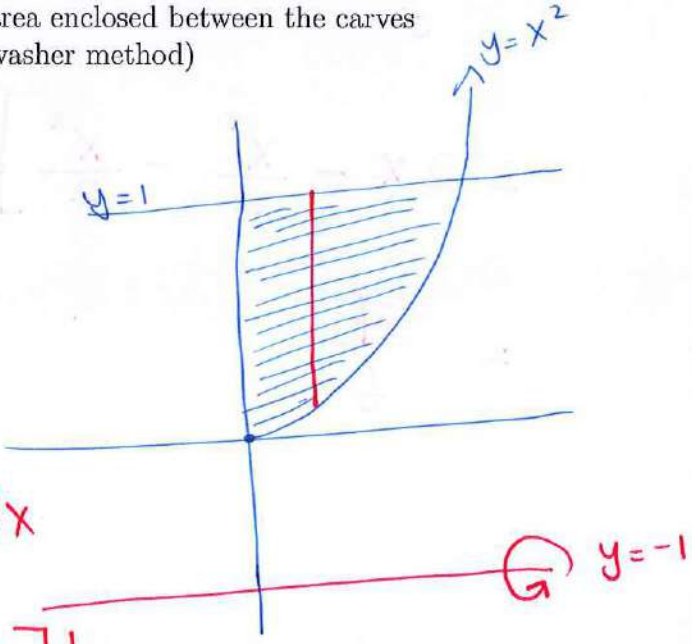


Section 6

BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT
MATH1411 - QUIZ 3

Name :..... Student Number..... Section 10/10

Question #1: 3.5 Pt.
Find the volume of solid generated by revolving the area enclosed between the carves $y = x^2$, y-axis , and $y = 1$ about the $y = -1$ (using washer method)



Washer Method

(1) $V = \pi \int_0^1 R^2(x) - r^2(x)\, dx$

(1) $= \pi \int_0^1 (2)^2 - (x^2+1)^2\, dx$

(1) $= \pi \int_0^1 4 - x^4 - 2x^2 - 1\, dx$

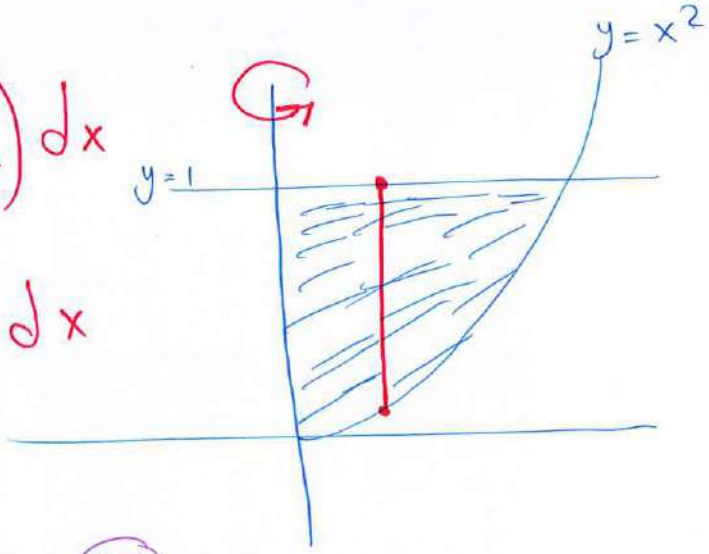
$= \pi \left[3x - \frac{x^5}{5} - \frac{2x^3}{3} \right]_0^1$

(as) $= \frac{32\pi}{15}$

Question #2:
Find the volume of solid generated by revolving the area enclosed between the carves $y = x^2$, y-axis , and $y = 1$ about the y-axis (using shell method) 3.5 Pt.

Shell Method

(1) $V = 2\pi \int_0^1 (\text{shell radius})(\text{shell height})\, dx$



(1) $= 2\pi \int_0^1 (x)(1 - x^2)\, dx$

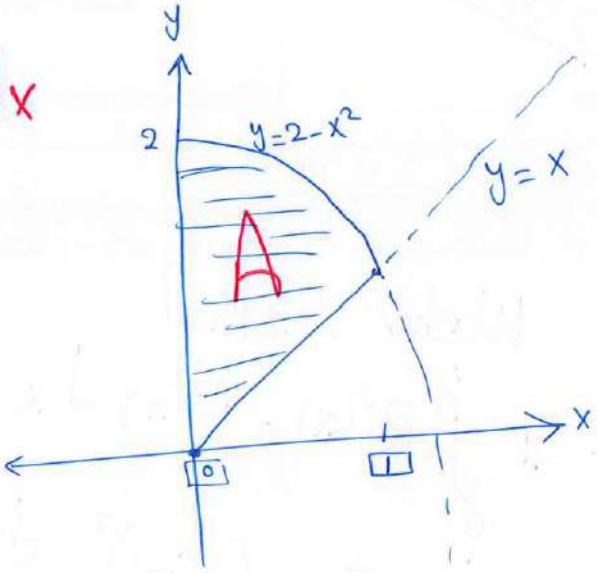
$= 2\pi \int_0^1 x - x^3\, dx$

(1) $= 2\pi \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \boxed{\frac{\pi}{2}}$ (0.5)

3 pt.

Question #3:

Find the area enclosed between the curve $y = 2 - x^2$, the y-axis, and the line $y = x$



(1) $\text{Area} = \int_0^1 (2 - x^2) - x \, dx$

(1) $= 2x - \frac{x^3}{3} - \frac{x^2}{2} \Big]_0^1$

(1) $= \frac{7}{6}$.

$f(x) = g(x)$

$2 - x^2 = x$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

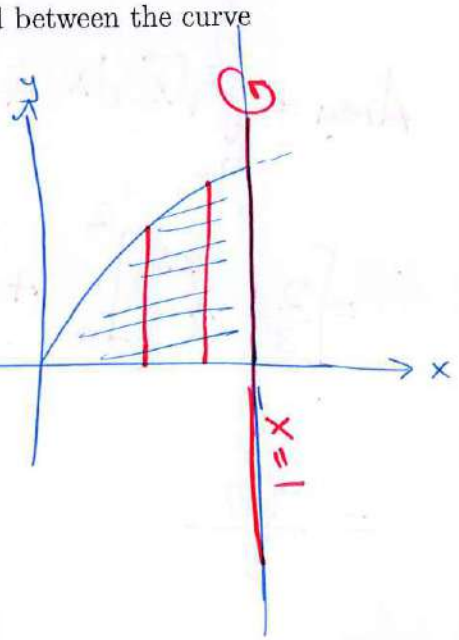
$x = -2$ or $\boxed{x = 1}$

Section (1)

BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT
MATH1411 - QUIZ 3

Name :..... Student Number..... Section 10/10

Question #1: 3.5 Pt.
Find the volume of the solid generated by revolving the region enclosed between the curve $y = 15\sqrt{x}, y = 0, x = 1$ about the line $x = 1$ (using Shell Method)



(1) $V = 2\pi \int_0^1 (\text{shell radius})(\text{shell height})\, dx$

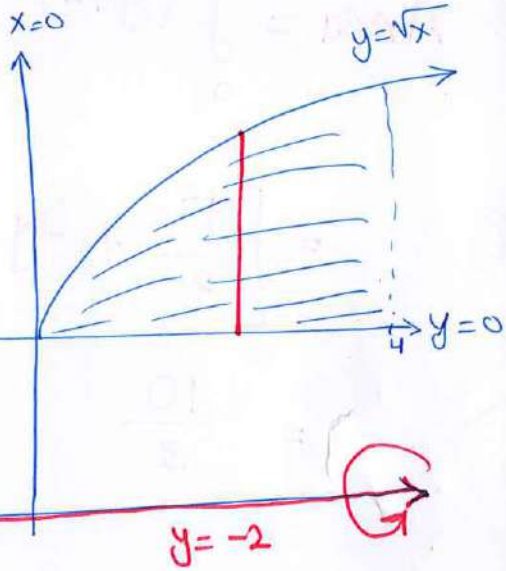
(1) $= 2\pi \int_0^1 (1-x)(15\sqrt{x})\, dx$

(1) $= 2\pi \left[15\left(\frac{2}{3}\right) x^{3/2} - 15\left(\frac{2}{5}\right) x^{5/2} \right]_0^1$

(0.5) $= 8\pi$

Question #2: 3.5 pt.
Find the volume of solid generated by revolving the region enclosed between the carves $y = \sqrt{x}$, x-axis, $0 \le x \le 4$ about the $y = -2$ using washer method (Don't Evaluate Integral)

Washer Method :-



(1) $V = \pi \int_0^4 R^2(x) - r^2(x)\, dx$

(1) $= \pi \int_0^4 (\sqrt{x}+2)^2 - (2)^2\, dx$

(1) $= \pi \int_0^4 x + 4\sqrt{x}\, dx$

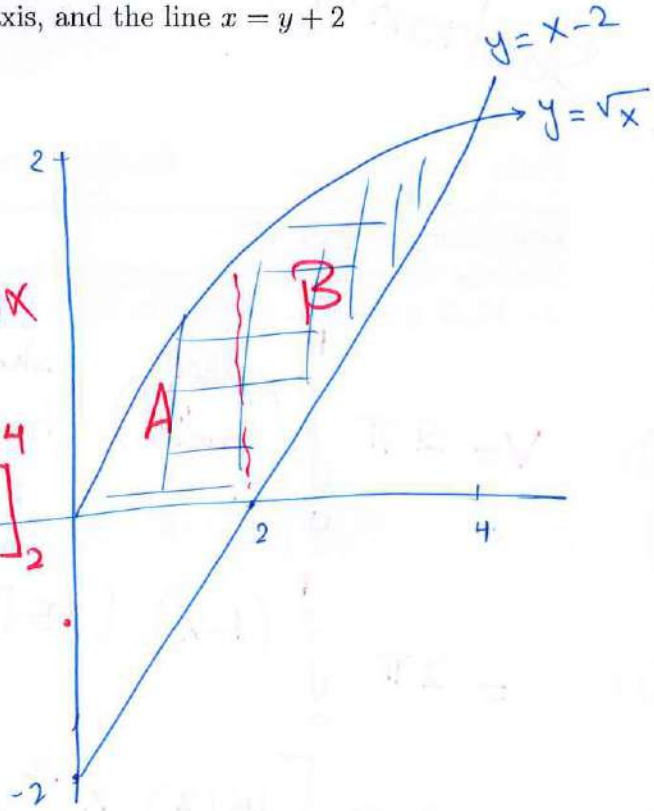
Question #3: 3 pt.

Find the area enclosed between the curve $x = y^2$, the x-axis, and the line $x = y + 2$

$$\text{Area} = \int_0^2 \sqrt{x}\,dx + \int_2^4 \sqrt{x} - (x-2)\,dx$$

$$= \left[\frac{2}{3}x^{3/2}\right]_0^2 + \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2} + 2x\right]_2^4$$

$$= \frac{10}{3}$$



OR

① $$\text{Area} = \int_0^2 (y+2) - y^2\,dy$$

① $$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_0^2$$

① $$= \frac{10}{3}$$